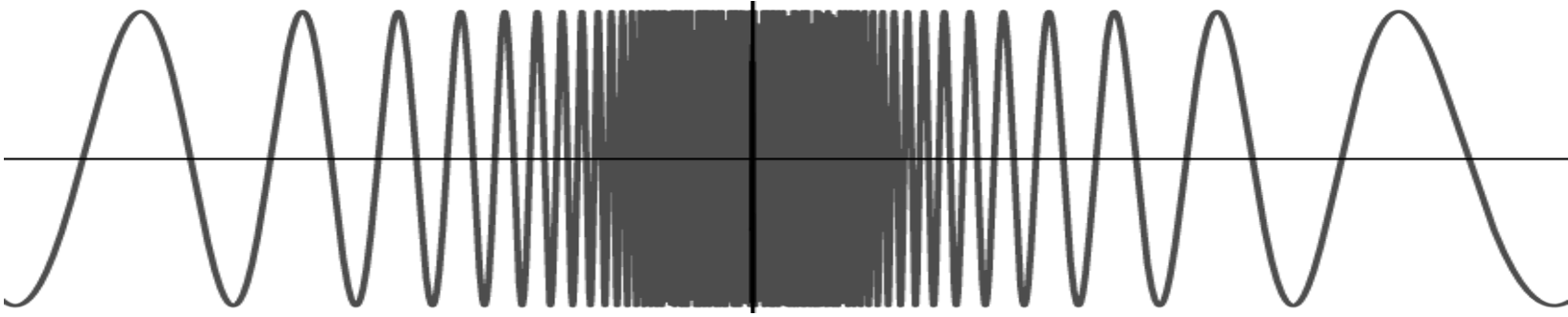

Thoughts on Teaching Mathematics

By Dr. Jeremy Brazas

The struggle is real... and necessary

During my graduate studies at the University of New Hampshire, my office was located next to that of a lecturer named Yitang Zhang who went by “Tom.” Tom was a quiet, kind, and fairly eccentric faculty member who could regularly be found walking up and down the halls with his hands behind his back or holding a cup of tea in one hand. On these walks, he’d be thinking - constantly thinking. On most days you could find Tom pacing the halls of the Mathematics Department like this. On such a walk, he might stop next to a classroom for a moment, look inside to see what was happening, and then continue his pacing... and thinking. Thinking about what? I didn't ask, so I suppose I didn't really know at the time.

For seven years after finishing his Ph.D., Tom struggled to find an academic position. To make a living, he worked various jobs for friends including a job at a Subway restaurant. But Tom didn’t give up. In 1999, he caught a break. A friend helped him secure a teaching position at the University of New Hampshire; there he lectured calculus and the occasional number theory course.

In 2014, the mathematics world exploded with news that someone had proved the Weak Twin Prime Conjecture. This foundational conjecture in number theory had remained unproven for hundreds of years. Surely to solve such a problem, that person must already be a well-established and well-respected number theorist, right? But it turned out that the solution actually came from an unheard of mathematician: Tom Zhang. I suppose I now know what Tom was pondering as he took his regular walks. Apparently, Tom had been working on and struggling with this problem for several years. No one ever expected him to solve it. Yet despite the difficulty and the many failures along the way, he never gave up.

Certainly, Tom Zhang is talented. There is no denying that. Natural talent, however was not the main reason he went from being an unheard of mathematician to appearing in news stories around the world. The key was **perseverance**. Even in my own research, I struggle with problems for weeks, months, and sometimes years. The successes I have are due to my willingness to fight, to struggle, and to be frustrated sometimes to the point of anger. The struggle is real and it is necessary. One of the most important components of my philosophy of teaching is to regularly emphasize this type of **self-patience**. Students often come to me frustrated or even crying because they can't understand something despite great time and effort. I view it as my job to tell them to be kind and patient with themselves. Learning new mathematics at any level, even at the research level, can be a painful battle. Shortcuts are in rare supply. Those who can persevere through the struggles are the ones who make significant progress and contributions to their respective fields. It's easy to forget this if the content you are teaching is old news to you. Good mathematics teaching requires that you recall the struggle and apply empathy.

The value of independent mathematical thinking

As an instructor, teaching a student how to think and grow independently is the greatest achievement I can possibly hope for. In teaching mathematics, it is tempting and all too easy to let the content and course structure become routine. From an instructional standpoint, very little about the mathematics changes from semester to semester. Science will forever improve on itself and the humanities change with the times, but the rigidity of mathematics is enduring. Despite this constancy, the students in my classes come and go. Every person that sits down apprehensively on the first day and hands in their final exam on the last has their own personal interests, dreams, and struggles. To maintain an awareness and sensitivity to this reality is crucial for successful teaching, especially in courses as foundational and difficult as calculus.

In my classroom, I seek to impart upon my students the same passion for the beauty and creativity inherent to mathematics that I developed during my own studies. This forms the framework for my dynamic and sometimes animated lectures. Students who see an instructor excited by the mathematics are much more likely to actually become interested in the content. Additionally, when students actually value mathematical thinking, they will be much more likely to struggle through challenging concepts and are more likely to **apply formal reasoning to other aspects of their lives**. I want students to walk away from my class with critical thinking experience that is applicable beyond the scope of the course content. To this end, I regularly pose challenging problems and get students to think about the problem solving process itself in addition to learning

traditional techniques.

An honest teacher will own up to the same thing

The creative process of communicating significant mathematical concepts to others is something that is developed over time. When I first began teaching, I thought I was pretty hot stuff – a natural. Years later, of course, I see that I was sorely mistaken. An honest teacher will own up to the same thing. Just as with the process of learning mathematics itself, there are no shortcuts to becoming a good teacher. If you are just looking to get the job done and you are not genuinely and actively looking to improve and grow, you probably aren't as good as you think you are. Our students deserve better! For me, it took years of lecturing and interacting with students to hone my oratory skills, my ability to know ahead of time what students will struggle with, and to know what concepts require extra explanation and visualization. I still seek to improve by finding new ways to capture student interest and apply new computational software.

If you are a young teacher who wants to improve, find a passionate and experienced teacher of a lower-level course who respects and is respected by their students. Attend their lectures: sit, listen, and watch. Ask other instructors how they deal with tricky classroom management situations. Take it all in and make it your own.

One does not create mathematical thinkers by doing mathematics

Conducting mathematics research (for me, in topology) is something I will do for many years simply because I love it. Research keeps the mind actively engaged in high level mathematics. It informs my teaching and offers students a glimpse into higher mathematical worlds. While research is rewarding in the sense that it allows one to push the boundaries of what is known a tiny bit further, it is almost always unclear (even in applied mathematics) when certain mathematical advancements will affect society if they do at all. Mathematics needs to be done, but one does not create mathematical thinkers by doing mathematics. The greatest impact factor a mathematician can achieve is through his or her students. The pedagogy we choose and the way we act in the classroom can have an enormous impact on the lives of hundreds or thousands of real people.

The practical stuff

Teaching in a classroom setting gives me the opportunity to make mathematics accessible and meaningful, encourage both academic and non-

academic maturity in students, and to design effective curriculum. Over the past several years, my preferred method of instruction has evolved into a combination of **traditional lecture and cooperative learning**. Students learn best when they receive scaffolding for problem solving and can make sense of concepts on their own. I have often implemented this philosophy at Georgia State by allowing students in-class time to practice solving problems while I am personally available to them (in-class, office hours, and extra review sessions). Building a strong rapport with my students has allowed me to regularly implement this highly interactive approach to teaching.

When teaching a class of fewer than 20 students (e.g. honors calculus), I like to give students the opportunity to learn in a cooperative workshop environment. This is a setting in which students, with guidance, can make computations, derive results, and explore new ideas in groups. One of the advantages of this approach is that students leave with a sense of ownership of the content. Additionally, students are more likely to master concepts when they are given the chance to communicate what they understand.

In a large lecture course, where group-work is less practical, I like to create a forum for students to solve problems creatively by introducing a metacognitive perspective into lectures. Providing definitions, theorems, and working through computations is important, but will not take students to the highest levels of learning. When introducing a new concept or working through a problem in lecture, I describe aloud the type of thinking that might lead one to a definition or to the next step in a solution. Encouraging students to model this behavior gets them thinking about how they might apply their experiences to solve new problems. This approach can make class more enjoyable for students since it is a reminder that mathematics is an evolving human enterprise, a fact often hidden by the formal nature of mathematics curriculum.

In any classroom setting, creating a rapport with students is crucial. I naturally maintain a high level of energy in-class and work hard to make sure students feel comfortable approaching me with questions and comments. Most importantly, my students know I sincerely wish for them to succeed in their goals. Exhibiting consistent encouragement and patience allows students to see I understand the struggles and frustrations that arise in learning mathematics and am there to support them. After all, learning new mathematics is challenging even at the research level. This part of my philosophy of teaching is particularly important for reaching students that lack strong backgrounds and are quickly turned off to mathematical challenges.