## MAT 162—Exam #1-9/26/13

Name: \_\_\_\_\_

Show all work using correct mathematical notation. Calculators are not allowed.

1. (10 points) Find the average value of the function  $f(x) = e^{2x}$  on the interval [0,3].

2. (15 points) Sketch the region bounded by the parabolas  $y = x^2$  and  $y = 2 - x^2$ , and find its area.

3. (10 points) The mass density (in kg/m) of a rod lying along the interval  $0 \le x \le 2$  is given by  $\rho(x) = \frac{x^2}{x^3 + 1}$ . Find the total mass of the rod.

4. (15 points) Find the volume of the solid obtained by rotating the region in the first quadrant bounded by the curves  $y = x^3$  and y = 4x about the y-axis. Include a sketch of the region, along with a representative disk, washer, or shell.

- 5. (10 points) Consider the portion of the curve y = 4/x between the points (1,4) and (2,2). Set up (**but do not evaluate**) integrals that give
  - (a) the length of the curve

(b) the area of the surface obtained by revolving the curve about the x-axis

6. (15 points) A hemispherical tank of radius 8 meters is filled to a height of 7 meters with water, which weighs  $9800 \text{ N/m}^3$ . Set up (**but do not evaluate**) a definite integral that gives the work required to pump all the water out through a spout that extends 2 meters above the tank's top. Show a representative slice and identify your variable(s) on the sketch provided.

- 7. (15 points) Consider the region bounded by the curve  $y = 1 + \cos x$  and the lines y = 0, x = 0, and  $x = \pi/2$ . Set up (**but do not evaluate**) definite integrals that give the volumes of the solids obtained by rotating the region about the following axes. For each part, show a representative disk, washer, or shell on the sketch provided.
  - (a) the line y = -1

(b) the line x = 2

8. (10 points) A rectangular swimming pool is 50 feet long, 20 feet wide, 10 feet deep at one end, and zero depth at the other end. The pool is full of water, which weighs 62.4 lb/ft<sup>3</sup>. Set up (**but do not evaluate**) a definite integral that gives the hydrostatic force on one of the triangular sides of the pool. Show a representative strip and identify your variable(s) on the sketch provided.