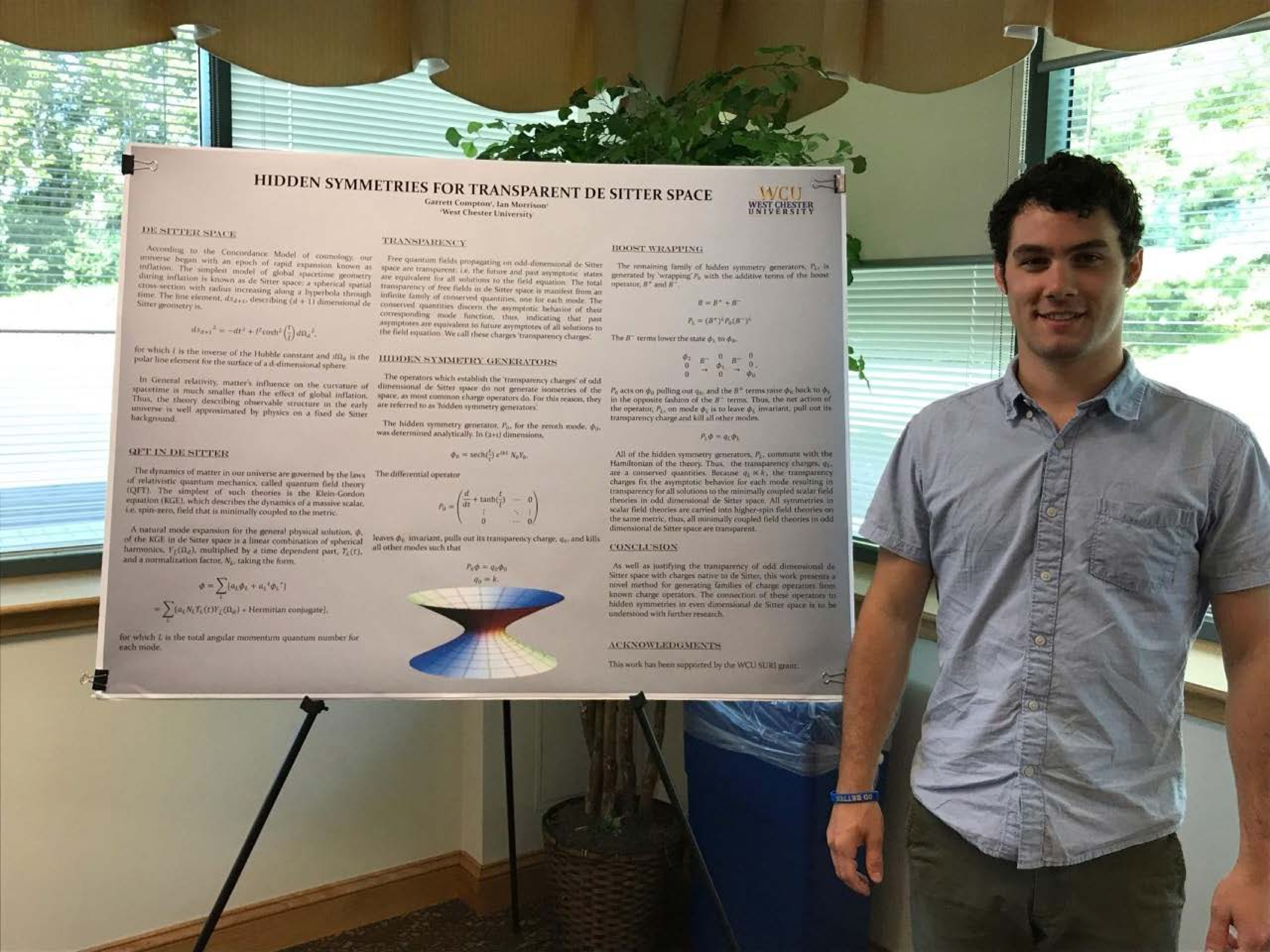
HIDDEN SYMMETRIES FOR TRANSPARENT DE SITTER SPACE
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DE SITTER SPACE
According to the Concordance Model of cosmology, our universe began with an epoch of rapid expansion known as inflation. The simplest model of global spacetime geometry during inflation is known as de Sitter space: a spherical spatial cross-section with radius increasing along a hyperbola through time. The line element, ds_{d+1}, describing (d + 1) dimensional de Sitter geometry is,
$ds_{d+1}^2 = -dt^2 + l^2 \cosh^2\left(\frac{t}{l}\right) d\Omega_d^2$,
for which l is the inverse of the Hubble constant and $d\Omega_d$ is the polar line element for the surface of a d-dimensional sphere.
In General relativity, matter's influence on the curvature of spacetime is much smaller than the effect of global inflation. Thus, the theory describing observable structure in the early universe is well approximated by physics on a fixed de Sitter background.
QFT IN DE SITTER
The dynamics of matter in our universe are governed by the laws of relativistic quantum mechanics, called quantum field theory (QFT). The simplest of such theories is the Klein-Gordon equation (KGE), which describes the dynamics of a massive scalar, i.e. spin-zero, field that is minimally coupled to the metric.
A natural mode expansion for the general physical solution, ϕ, of the KGE in de Sitter space is a linear combination of spherical harmonics, $Y_{\vec{L}}(\Omega_d)$, multiplied by a time dependent part, $T_L(t)$, and a normalization factor, N_L, taking the form,
$\phi = \sum_L [a_L \phi_L + a_L^\dagger \phi_L^*]$
$= \sum_L [a_L N_L T_L(t) Y_{\vec{L}}(\Omega_d) + \text{Hermitian conjugate}]$,
for which L is the total angular momentum quantum number for each mode.
TRANSPARENCY
Free quantum fields propagating on odd-dimensional de Sitter space are transparent: i.e. the future and past asymptotic states are equivalent for all solutions to the field equation. The total transparency of free fields in de Sitter space is manifest from an infinite family of conserved quantities, one for each mode. The conserved quantities discern the asymptotic behavior of their corresponding mode function, thus, indicating that past asymptotes are equivalent to future asymptotes of all solutions to the field equation. We call these charges 'transparency charges'.
HIDDEN SYMMETRY GENERATORS
The operators which establish the 'transparency charges' of odd dimensional de Sitter space do not generate isometries of the space, as most common charge operators do. For this reason, they are referred to as 'hidden symmetry generators'.
The hidden symmetry generator, P_0, for the zeroth mode, ϕ_0, was determined analytically. In (2+1) dimensions,
$\phi_0 = \text{sech}\left(\frac{t}{l}\right) e^{\pm it} N_0 Y_0$.
The differential operator
$P_0 = \begin{pmatrix} \frac{d}{dt} + \tanh\left(\frac{t}{l}\right) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$
leaves ϕ_0 invariant, pulls out its transparency charge, q_0, and kills all other modes such that
$P_0 \phi = q_0 \phi_0$
$q_0 = k$.
BOOST WRAPPING
The remaining family of hidden symmetry generators, P_L, is generated by 'wrapping' P_0 with the additive terms of the boost operator, B^+ and B^-.
$B = B^+ + B^-$
$P_L = (B^+)^L P_0 (B^-)^L$
The B^- terms lower the state ϕ_L to ϕ_0.
$\begin{matrix} \phi_2 \\ 0 \\ 0 \end{matrix} \xrightarrow{B^-} \begin{matrix} 0 \\ \phi_1 \\ 0 \end{matrix} \xrightarrow{B^-} \begin{matrix} 0 \\ 0 \\ \phi_0 \end{matrix}$.
P_0 acts on ϕ_0 pulling out q_0, and the B^+ terms raise ϕ_0 back to ϕ_L in the opposite fashion of the B^- terms. Thus, the net action of the operator, P_L, on mode ϕ_L is to leave ϕ_L invariant, pull out its transparency charge and kill all other modes.
$P_L \phi = q_L \phi_L$
All of the hidden symmetry generators, P_L, commute with the Hamiltonian of the theory. Thus, the transparency charges, q_L, are a conserved quantities. Because $q_L \propto k_L$ the transparency charges fix the asymptotic behavior for each mode resulting in transparency for all solutions to the minimally coupled scalar field theories in odd dimensional de Sitter space. All symmetries in scalar field theories are carried into higher-spin field theories on the same metric, thus, all minimally coupled field theories in odd dimensional de Sitter space are transparent.
CONCLUSION
As well as justifying the transparency of odd dimensional de Sitter space with charges native to de Sitter, this work presents a novel method for generating families of charge operators from known charge operators. The connection of these operators to hidden symmetries in even dimensional de Sitter space is to be understood with further research.
ACKNOWLEDGMENTS
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